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SUPER-POINCARÉ COVARIANT CANONICAL FORMULATION OF SUPERPARTICLES AND GREEN-SCHWARZ SUPERSTRINGS *

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ABSTRACT

First, a new unified covariant formulation simultaneously describing both superparticles and spinning particles is proposed. In this formulation both models emerge as different gauge fixings from a more general point-particle model with larger gauge invariance. The general model possesses covariant and functionally independent first-class constraints only. Next, the above construction is generalized to the case of Green-Schwarz (GS) superstrings. This allows straightforward application of the Batalin-Fradkin-Vilkovisky (BFV) Becchi-Kouet-Stora-Tyutin (BRST) formalism for a manifestly super-Poincare covariant canonical quantization. The cooresponding BRST charge turns out to be remarkably simple and is of rank one. It is used to construct a covariant BFV hamiltonian for the GS superstring exhibiting explicit Parisi-Sourlas OSp(1,1/2) symmetry.

1. INTRODUCTION AND MOTIVATION.

In view of the importance of manifest space-time supersymmetry for the superstring theory (anomaly cancellation, finiteness, vanishing cosmological constant etc.; see [1] for a detailed discussion and a long list of references), much interest was attracted to the super Poincare co-covariant Green-Schwarz (GS) superstring [2].

In its original formulation [2] the GS superstring exhibits some serious problems which obstructed for a long time the progress towards its consistent super-Poincare covariant quantization :

(i) The original GS action (see Eqs.(25)-(27) below) defines a constrained system containing complicated mixture of first and second-class fermionic constraints (Eqs.(27) and (30)) which cannot be separated in a Lorentz covariant way [3]. The covariant separation proposed in ref.[4] leads in fact to functionally dependent (reducible, according to the terminology of Batalin-fradkin-Vilkovisky (BFV) [5]) sets of constraints. Application of the correct BFV procedure [5] to treat these reducible constraints would either break Lorentz invariance or would force the level of reducibility (i.e. the number of generations of ghosts for ghosts) to be infinite [6].

(ii) The presence of second-class constraints (even after covariant and irreducible disentangling with the help of the harmonic variables introduced in [7,8]) leads to highly complicated Dirac brackets among the superstring coordinates $X^{\mu}, \Theta_{A\alpha}$. Thus, the initial connection with the geometry of the embedding superspace is lost.

(iii) Siegel's superstring [9], which is physically equivalent to the GS superstring [10], contains covariant first-class constraints only. However, these constraints once again form a reducible set with an infinite level of reducibility [6]. The formalism, proposed in [11] to eliminate the higher ghost generations within the BFV treatment of the Siegel's superstring, explicitly breaks Lorentz invariance since it introduces constant light-like Lorentz vectors in the BFV-BRST action which are not dynamical degrees of freedom.(Similar explicit breaking of Lorentz invariance occurs also in the quantization scheme for superparticles proposed in [12].)

The problem (i) above was solved in [7,8] (see Eqs.(31) below) by extending the usual D=10 superspace to include additional bosonic harmonic coordinates with a simple geometrical meaning (Eqs. (3),(4) below). Next, in order to achieve simple canonical Dirac brackets among the superstring coordinates, we had to impose in ref.[8] covariant gauge-fixing for the fermionic $\varkappa$-gauge invariance (the first-class part of the fermionic GS constraints). Thus, although acting in a manifestly Lorentz covariant way, the space-time translations and supersymmetry transformations became nonlinearly realized.

In the present report we propose a new physically equivalent reformulation of the GS superstring by embedding it into a larger system with additional gauge invariance which contains besides the bosonic harmonic coordinates of [7,8] additional fermionic string variables (cf. also ref.[13]). The most important property of the enlarged superstring system is that it possesses

covariant and irreducible first-class constraints only. Thus, the covariant BFV-BRST quantization procedure [5] may be directly applied and the whole super-Poincare invariance is manifestly preserved.

The whole procedure is first illustrated at the instance of massless superparticles [14]. In particular, we get a generalized point-particle action with large gauge invariance which yields unified covariant description simultaneously of both superparticles and spinning particles [15]. Both models emerge as different gauge-fixings from the above generalized action.

We compute the BRST charge Q_{BRST} of the new enlarged GS superstring with turns out to be particularly simple and is of rank one. The latter means that Q_{BRST} does not contain higher order ghost terms. This property is to be contrasted with the result of ref.[13] where Q_{BRST} was found to be of rank two. There is, however, no contradiction between both results : according to the general BFV theory [5] the rank of Q_{BRST} is not an invariant notion. The rank can always be changed by taking suitable functionally independent combinations of the first-class constraints with coefficients depending on the canonical variables. However, the freedom in choosing the independent combinations of constraints and, therefore, the possibility of changing the rank are severely restricted by physical requirements such as preserving global symmetries (super-Poincare in our case) etc.

The implications of this general scheme for the GS superstring are as follows. In Section 3 we succeed in finding suitable combinations $\widehat{\mathcal{D}}_A^{\alpha}$ (Eq.(32)) of the first-class fermionic constraints $\widehat{D}_A^{+1/2a}$, $\widehat{G}_A^{+1/2a}$ (Eq.(32')) from ref.[13] in such a way that the rank of Q_{BRST} reduces from two to one by preserving simultaneously super-Poincare invariance.

Finally, we use Q_{BRST} to construct the BFV hamiltonian for the GS superstring which exhibits both manifest space-time super symmetry as well as explicit Parisi-Sourlas [16] OSp(1,1/2) symmetry (cf. refs.[17-19]).

Throughout this report we shall always work in the hamiltonian (phase-space) formalism. Also, we take for definiteness the space-time dimension D=10 although the present construction can be straightforwardly extended to other dimensions (D=3,4,6) where classical GS superstrings exist [2].

2. UNIFIED DESCRIPTION OF SUPERPARTICLES AND SPINNING PARTICLES.

The standard form of the superparticle action [14] in (N extended D = 10 superspace $(x^\mu, \theta_{A\alpha})$, A=1,...,N, reads :

$$S_{super} = \int d\tau \Big[p_\mu \partial_\tau x^\mu + \sum_A p^\alpha_{\theta A} \partial_\tau \theta_{A\alpha} - \Lambda p^2 - \sum_A \Lambda_{A\alpha} D^\alpha_A \Big]. \qquad (1)$$

In (1) $\theta_{A\alpha}$ are left-handed Majorana-Weyl (MW) spinors, Λ, $\Lambda_{A\alpha}$ are Lagrange multipliers, and the fermionic constraints :

$$D^\alpha_A = -i p^\alpha_{\theta A} - \not p^{\alpha\beta} \theta_{A\beta} \; ,$$
$$\{ D^\alpha_A, D^\beta_B \}_{PB} = 2i \delta_{AB} \not p^{\alpha\beta} \; , \qquad (2)$$

form a mixture of 8N first-class and 8N second-class constraints on the constraint surface $p^2=0$. In (2) $\{\ ,\ \}_{PB}$ denotes graded Poisson brackets.

Let us now introduce as in [7,8] the D=10 harmonic N-superspace $(x^\mu, \theta_{A\alpha}; v^{\pm 1/2}_\alpha, u^a_\mu)$ where :

(i) $v^{\pm 1/2}_\alpha$ are two D=10 left-handed MW bosonic spinors,

(ii) u^a_μ are eight $(a=1,\ldots,8)$ D=10 Lorentz vectors,

which satisfy the kinematical constraints :

$$(v^{+1/2} \sigma_\mu v^{+1/2})(v^{-1/2} \sigma^\mu v^{-1/2}) = -1 \; ,$$
$$u^a_\mu (v^{\pm 1/2} \sigma^\mu v^{\pm 1/2}) = 0 \quad , \quad u^a_\mu u^{b\mu} = C^{ab} \; . \qquad (3)$$

The group SO(8)xSO(1,1) acts on u^a_μ, $v^{\pm 1/2}_\alpha$ as an internal group of local rotations where u^a_μ belong to any one of the three inequivalent 8-dimensional representations ((v),(s),(c)) of SO(8), whereas $v^{\pm 1/2}_\alpha$ carry charge $\pm 1/2$ under SO(1,1). In the last line of (3) C^{ab} denotes the invariant metric tensor in the relevant SO(8) representation space (C^{ab} is the unit matrix for (v) and it is the left(right) chiral charge conjugation matrix for (s), (c), respectively).

Let us recall, that the composite Lorentz-vectors :

$$u^\pm_\mu = v^{\pm 1/2} \sigma_\mu v^{\pm 1/2} \qquad (4)$$

are identically light-like due to the well-known D=10 Fierz identity (see e.g. [1]) :

$$(\sigma_\mu)^{\alpha\beta} (\sigma^\mu)^{\gamma\delta} + (\sigma_\mu)^{\beta\delta} (\sigma^\mu)^{\alpha\delta} + (\sigma_\mu)^{\gamma\alpha} (\sigma^\mu)^{\beta\delta} = 0 \; . \qquad (5)$$

Thus, the vectors u^a_μ (3) together with $u^\pm_\mu$ (4) realize the coset space SO(1,9)/SO(8)xSO(1,1) (cf. refs.[20]).

The action, involving the harmonics only, reads [7,8]:

$$S_{harmonic} = \int d\tau \Big[p^\mu_{ua} \partial_\tau u^a_\mu + p_v^{-1/2\alpha} \partial_\tau v^{+1/2}_\alpha + p_v^{+1/2\alpha} \partial_\tau v^{-1/2}_\alpha$$
$$- \Lambda_{ab} d^{ab} - \Lambda^{+-} d^{-+} - \Lambda^-_a d^{+a} \Big] \; . \qquad (6)$$

Because of the kinematical constraints (3), the harmonic momenta are similarly kinematically constrained :

$$p^{\mu(a}_u u^{b)}_\mu = 0 \quad , \quad p^a_{u\mu} (v^{\pm 1/2} \sigma^\mu v^{\pm 1/2}) = 0 \quad , \quad v^{+1/2}_\alpha p_v^{-1/2\alpha} + v^{-1/2}_\alpha p_v^{+1/2\alpha} = 0 .$$

In (6) Λ_{ab}, Λ^{+-}, Λ^{-a} are Lagrange multipliers and the corresponding first-class constraints read (in first-quantized form) :

$$D^{ab} = u^a_\mu \partial/\partial u_{\mu b} - u^b_\mu \partial/\partial u_{a\mu} \qquad \text{(SO(8) generators)} \qquad (7)$$

$$D^{-+} = 1/2 \left(v^{+1/2}_\alpha \partial/\partial v^{+1/2}_\alpha - v^{-1/2}_\alpha \partial/\partial v^{-1/2}_\alpha \right) \qquad \text{(SO(1,1) generator)} \qquad (8)$$

$$D^{+a} = u^+_\mu \partial/\partial u_{\mu a} + 1/2 \left(v^{-1/2} \sigma^+ \sigma^a \partial/\partial v^{-1/2} \right) \qquad (9)$$

(half of the coset generators corresponding to SO(1,9)/SO(8)xSO(1,1)).

Here and below the following short-hand notations will be used :

$$A^\pm \equiv u^\pm_\mu A^\mu = v^{\pm 1/2} \not A v^{\pm 1/2} \quad , \quad A^a \equiv u^a_\mu A^\mu \qquad (10)$$

for any Lorentz vector A^μ. Let us particularly stress that $A^\pm$, A^a are Lorentz scalars and they should not be confused with vector components in the non-covariant light-cone formalism.

As already explained in detail in refs.[7,8], the harmonics $v^{\pm 1/2}_\alpha$, u^a_μ, whose dynamics is governed by (6), are pure-gauge degrees of freedom.

Finally, we introduce additional fermionic variables ψ^μ_A on the point-particle world line - the same which appear in the action for the spinning particle [15] :

$$S_{spinning} = \int d\tau \Big[p_\mu \partial_\tau x^\mu + i \sum_A \psi^\mu_A \partial_\tau \psi_{A\mu}$$
$$- \Lambda p^2 - \sum_A M_A (p_\mu \psi^\mu_A) \Big] \qquad (11)$$

Here, similarly, Λ and M_A are Lagrange multipliers for the first class constraints p^2, $p_\mu \psi^\mu_A$.

Now, let us consider the following generalized point-particle action :

$$S = \int d\tau \Big[p_\mu \partial_\tau x^\mu + \sum_A \big(p^\alpha_{\theta A} \partial_\tau \theta_{A\alpha} + i \psi^\mu_A \partial_\tau \psi_{A\mu} \big) - \Lambda p^2 - \sum_A \big(\Lambda_{A\alpha} \hat{\hat{D}}^\alpha_A + M_A J_A \big) \Big] + S_{harmonic} , \qquad (12)$$

where the last term is as in (6) and the fermionic constraints are given by (D^α_A as in (2)) :

$$\hat{\hat{D}}^\alpha_A \equiv D^\alpha_A - (p^+)^{-1/2} (\not p \sigma^\mu \sigma^+ v^{-1/2})^\alpha \psi_{A\mu} \equiv \hat{D}^\alpha_A + 2 (p^+)^{-1/2} (\not p v^{-1/2})^\alpha \psi^+_A , \qquad (13)$$

$$J_A \equiv p_\mu \psi^\mu_A . \qquad (14)$$

In the action (12) all constraints are irreducible and first-class only :

$$\{ \hat{\hat{D}}^\alpha_A , \hat{\hat{D}}^\beta_B \}_{PB} = i \delta_{AB} (\sigma^+)^{\alpha\beta} (p^+)^{-1} p^2 \ , \quad \{ J_A , J_B \}_{PB} = - i \delta_{AB} p^2 ,$$

$$\{ \hat{\hat{D}}^\alpha_A , J_B \}_{PB} = - i \delta_{AB} (\sigma^+ v^{-1/2})^\alpha (p^+)^{-1/2} p^2 \ ;$$

$$\{ d^{ab} , d^{cd} \}_{PB} = C^{bc} d^{ad} - C^{ac} d^{bd} + C^{ad} d^{bc} - C^{bd} d^{ac} ,$$

$$\{ d^{ab} , d^{+c} \}_{PB} = C^{bc} d^{+a} - C^{ac} d^{+b} , \qquad (15)$$

Let us remark at this point that according to (12) the canonical Poisson brackets (in fact - Dirac brackets) for ψ^μ_A read :

$$\{ \psi^\mu_A , \psi^\nu_B \}_{PB} = - i \eta^{\mu\nu} .$$

If desired, one can separate ψ^μ_A into canonically conjugated pairs by introducing besides the harmonics (3) a second generation of harmonics [7,8] :

$$w^\kappa_a w^{\ell a} = \overline{w}^{\dot\kappa}_a \overline{w}^{\dot\ell a} = 0 \ , \quad w^\kappa_a \overline{w}^{\dot\ell a} = C^{\kappa\dot\ell} , \qquad (16)$$

realizing the coset space SO(8)/SU(4)xU(1). Here $C^{\kappa\dot\ell}$ denotes the D=6 chiral charge conjugation matrix and under the group SU(4)x U(1) of local rotations $w^\kappa_a , \overline{w}^{\dot\kappa}_a$ transform as (4,1/2),(4,-1/2), respectively. With the help of (3) and (16) the canonically conjugated pairs of ψ^μ_A are given as :

$$\big(\psi^+_A = u^+_\mu \psi^\mu_A , \ \psi^\kappa_A = w^\kappa_a u^a_\mu \psi^\mu_A \big) , \ \big(\psi^-_A = u^-_\mu \psi^\mu_A , \ \overline{\psi}^{\dot\kappa}_A = \overline{w}^{\dot\kappa}_a u^a_\mu \psi^\mu_A \big) . \qquad (17)$$

The action (12) simultaneously describes both the superparticle defined by (1) and the spinning particle defined by (11) as

two different gauge choices. Indeed, let us first impose for the system (12) the gauge fixing conditions :

$$\theta_{A\alpha} = 0 \qquad (18)$$

corresponding to the constraints $\hat{\hat{D}}^\alpha_A = 0$ (13). Then, clearly, the system (12) reduces to $S_{spinning}$ (11) plus $S_{harmonic}$ (6) which, of course, is completely decoupled from (11).

Next, let us consider another gauge fixing condition for the action (12) (recall the covariant notations (10)) :

$$\psi^+_A = 0 \qquad (19)$$

corresponding to the constraint $J_A = 0$ (14) whereby ψ^-_A is expressed as :

$$\psi^-_A \Big|_{J_A = 0} = (p^+)^{-1} p_a \psi^a_A .$$

In the gauge (19) the action (12) reduces to :

$$\tilde{S}_{super} = \int d\tau \Big[p_\mu \partial_\tau x^\mu + \sum_A \big(p^\alpha_{\theta A} \partial_\tau \theta_{A\alpha} + i \psi^a_A \partial_\tau \psi_{Aa} \big) - \Lambda p^2 - \sum_A \Lambda_{A\alpha} \hat{D}^\alpha_A \Big] + S_{harmonic} . \qquad (20)$$

Note here the important fact that both S(12) and $\tilde{S}_{super}$(20) are manifestly invariant under (N-extended) supersymmetry transformations (as is also (1)) :

$$\delta_{SS} x^\mu = - i \sum_A \varepsilon_A \sigma^\mu \theta_A \ , \quad \delta_{SS} \theta_{A\alpha} = \varepsilon_{A\alpha} ,$$
$$\delta_{SS} \psi^a_A = \delta_{SS} v^{\pm 1/2}_\alpha = \delta_{SS} u^a_\mu = 0 . \qquad (21)$$

Let us recall that in the hamiltonian frame work the supersymmetry generators read :

$$Q^\alpha_A = - i p^\alpha_{\theta A} + \not p^{\alpha\beta} \theta_{A\beta} .$$

Each Lorentz spinor $\hat{D}^\alpha_A$ (13') (A=fixed) may be covariantly decomposed with the aid of the harmonics (3) into two SO(8) eight-component objects $\hat{D}_A^{+1/2 a}, \hat{G}_A^{+1/2 a}$ (cf. ref. (7,8)) :

$$\hat{D}_A^{\alpha} = (p^+)^{-1}(\sigma^a v^{+1/2})^{\alpha} \hat{D}_{Aa}^{+1/2} + (p^+)^{-1}(p\!\!/\sigma + \sigma^a v^{-1/2})^{\alpha} \hat{G}_{Aa}^{+1/2} ,$$

or inversely :

$$\hat{D}_A^{+1/2 a} = v^{+1/2}\sigma^a p\!\!/ \hat{D}_A = v^{+1/2}\sigma^a p\!\!/ D_A \qquad (22)$$

(the term containing ψ_A^a cancels out due to (5)) ;

$$\hat{G}_A^{+1/2 a} = \tfrac{1}{2}(v^{-1/2}\sigma^a\sigma + \hat{D}_A) = \tfrac{1}{2}(v^{-1/2}\sigma^a\sigma + D_A) + (p^+)^{1/2}\psi_A^a . \qquad (23)$$

Now, imposing further gauge fixing conditions in (20) :

$$\psi_A^a = 0 \qquad (24)$$

corresponding to the constraints $\hat{G}_A^{+1/2 a} = 0$ (23) the action (20) readily reduces to S_{super} (1) plus $S_{harmonic}$. The latter , of course, once again completely decouples from S_{super} .

The reason why we singled out $\tilde{S}_{super}$ (20) as an intermediate step in the gauge fixing procedure leading from (12) to (1) is that the super-Poincare invariant action (20) possesses covariant and irreducible first-class constraints only, unlike the standard superparticle action (1) (cf. discussion in Section 1). Therefore, $\tilde{S}_{super}$ (20) is an appropriate starting point for manifestly super Poincare covariant BFV-BRST quantization of the superparticle (1) (cf.refs.(7) for closely related approaches). Moreover, it is just the form of the action (20) which allows direct generalization to the case of GS superstrings in view of their super Poincare covariant quantization. This is the topic of the next Section.

3. ENLARGED HARMONIC GREEN-SCHWARZ SUPERSTRING. Here we shall discuss for definiteness the type IIB GS superstring (2,1). Its action, written in hamiltonian form, reads :

$$S_{GS} = \int d\tau \int_{-\pi}^{\pi} d\xi \left[\mathcal{P}_\mu \partial_\tau X^\mu + \sum_A p_{\theta A}^{\alpha} \partial_\tau \theta_{A\alpha} - \sum_A \left(\Lambda_A T_A + \Lambda_{A\alpha} \mathcal{D}_A^{\alpha} \right) \right] . \qquad (25)$$

Here $\theta_{A\alpha}$ (A=1,2) are two D=10 left-handed MW spinors. The action (25) is manifestly super-Poincare invariant (see (21)) where the supersymmetry generators in the hamiltonian framework read :

$$Q_A^{\alpha} = \int_{-\pi}^{\pi} d\xi\, Q_A^{\alpha}(\xi) ,$$

$$Q_A^{\alpha}(\xi) = -i p_{\theta A}^{\alpha} + \left[\mathcal{P}^\mu + (-1)^A \left(X'^\mu + i\theta_A \sigma^\mu \theta'_A \right) \right] (\sigma_\mu \theta_A)^{\alpha} .$$

The constraints in (25) are given explicitly as follows (4,10)

$$T_A \equiv \Pi_A^2 + 4i(-1)^A \mathcal{D}_A^{\alpha} \theta'_{A\alpha} = \qquad (26)$$

$$= \left(\mathcal{P} + (-1)^A X' \right)^2 - 4(-1)^A \theta'_{A\alpha} p_{\theta A}^{\alpha}$$

(reparametrization constraints) where $\Pi_A^\mu \equiv \mathcal{P}^\mu + (-1)^A [X'^\mu + 2i\theta_A \sigma^\mu \theta'_A]$;

$$\mathcal{D}_A^{\alpha} \equiv -i p_{\theta A}^{\alpha} - \left[\mathcal{P}^\mu + (-1)^A \left(X'^\mu + i\theta_A \sigma^\mu \theta'_A \right) \right] (\sigma_\mu \theta_A)^{\alpha} . \qquad (27)$$

Their Poisson bracket algebra reads :

$$\{ T_A(\xi), T_B(\eta) \}_{PB} = 8(-1)^A \delta_{AB} \left[T_A(\xi)\delta'(\xi-\eta) + \tfrac{1}{2} T'_A(\xi)\delta(\xi-\eta) \right] , \qquad (28)$$

$$\{ T_A(\xi), \mathcal{D}_B^{\alpha}(\eta) \}_{PB} = 4(-1)^A \delta_{AB}\, \mathcal{D}_A^{\alpha}(\xi)\delta'(\xi-\eta) , \qquad (29)$$

$$\{ \mathcal{D}_A^{\alpha}(\xi), \mathcal{D}_B^{\beta}(\eta) \}_{PB} = 2i\delta_{AB}\, \delta(\xi-\eta)\, \Pi\!\!\!/_A(\xi) . \qquad (30)$$

As it is clear from (30) and (26), the fermionic constraints $\mathcal{D}_A^{\alpha}$ (27) are a mixture of first-class constraints (generators of the fermionic $\varkappa$-gauge invariance (2,1)) and of second-class constraints. In ref.(8) the first-class parts of $\mathcal{D}_A^{\alpha}$ were covariantly disentangled by means of the harmonics (3) :

$$\mathcal{D}_A^{+1/2 a} = v^{+1/2}\sigma^a \Pi\!\!\!/_A \mathcal{D}_A \qquad \text{(first-class; cf. (22))} ,$$

$$G_A^{+1/2 a} = \tfrac{1}{2}\left(v^{-1/2}\sigma^a\sigma + \mathcal{D}_A \right) \qquad \text{(second-class; cf. (23))} . \qquad (31)$$

Here we propose another route to covariantly quantize the GS superstring (25) by generalizing the superparticle action (20). Let us repeat our warning to the reader that according to the definitions (10) the objects $A^{\pm}, A^{a}$ are Lorentz scalars and do not represent vector components of A^{μ} in the non-covariant light-cone formalism.

Introducing the harmonics $v_{\alpha}^{\pm 1/2}, u_{\mu}^{a}$ (3) and additional fermionic string variables $\Psi_A^{a}(\xi)$ satisfying :

$$\{ \Psi_A^{a}(\xi), \Psi_B^{b}(\eta) \}_{PB} = -i\delta_{AB} C^{ab} \delta(\xi-\eta) ,$$

one can easily construct the string analogue of (13) as purely first-class generalization of the fermionic constraints $\mathcal{D}_A^\alpha$ (27):

$$\hat{\mathcal{D}}_A^\alpha = \mathcal{D}_A^\alpha - 2i(-1)^A(\Pi_A^+)^{-1}(\sigma_a v^{+1/2})^\alpha \mathcal{R}_A^{+1/2abc}\Psi_{Ab}\Psi_{Ac} + \\ + (\Pi_A^+)^{-1/2}(\not{\Pi}_A\sigma + \sigma^a v^{-1/2})^\alpha \Psi_{Aa} . \qquad (32)$$

Here the following notations is used :

$$\mathcal{R}_A^{+1/2abc} \equiv (v^{+1/2}\sigma^a\sigma^d\sigma^c v^{-1/2})(v^{-1/2}\sigma^b\sigma_d\sigma^+\theta_A') .$$

Let us note the connection of $\hat{\mathcal{D}}_A^\alpha$ (32) with the irreducible fermionic first-class constraints $\hat{D}_A^{+1/2a}, \hat{G}_A^{+1/2a}$ of ref. [13] :

$$\hat{\mathcal{D}}_A^\alpha = (\Pi_A^+)^{-1}(\sigma^a v^{+1/2})^\alpha \hat{D}^{+1/2}_{A\,a} + (\Pi_A^+)^{-1}(\not{\Pi}_A\sigma + \sigma^a v^{-1/2})\hat{G}^{+1/2}_{Aa} \qquad (32')$$

where

$$\hat{D}_A^{+1/2a} = D_A^{+1/2a} - 2i(-1)^A\mathcal{R}_A^{+1/2abc}\Psi_{Ab}\Psi_{Ac} ,$$

$$\hat{G}_A^{+1/2a} = G_A^{+1/2a} + (\Pi_A^+)^{1/2}\Psi_A^a ,$$

with $D_A^{+1/2a}, G_A^{+1/2a}$ defined in (31).

In order to preserve the first-class property of the Poisson bracket relations (28),(29), we have to modify simultaneously T_A :

$$\hat{T}_A(\xi) \equiv T_A(\xi) + 2i(-1)^A\Psi_A^a(\xi)\Psi_{Aa}'(\xi) = \\ = (\mathcal{P} + (-1)^A X')^2 - 4i(-1)^A\theta_{A\alpha}'\delta/\delta\theta_{A\alpha} + 2i(-1)^A\Psi_A^a\Psi_{Aa}' . \qquad (33)$$

Thus, we arrive at the following enlarged harmonic GS superstring action :

$$\tilde{S}_{GS} = \int d\tau \int_{-\pi}^{\pi} d\xi \Big[\mathcal{P}_\mu \partial_\tau X^\mu + \sum_A \big(p_{\theta A}^\alpha \partial_\tau \theta_{A\alpha} + i\Psi_A^a\partial_\tau\Psi_{Aa}\big) \\ - \sum_A\big(\Lambda_A\hat{T}_A + \Lambda_{A\alpha}\hat{\mathcal{D}}_A^\alpha\big)\Big] + \hat{S}_{harmonic} . \qquad (34)$$

Here $\hat{S}_{harmonic}$ is of the same form as the action (6) with the harmonic constraints d^{ab} (7) (generators of the SO(8) rotations) modified to :

$$\hat{d}^{ab} = d^{ab} + i\sum_A \int_{-\pi}^{\pi} d\xi\, \Psi_A^a(\xi)\Psi_A^b(\xi) .$$

The algebra of the constraints in the action (34), which are now all irreducible and first-class, takes the form :

$$\{\hat{\mathcal{D}}_A^\alpha(\xi), \hat{\mathcal{D}}_B^\beta(\eta)\}_{PB} = i\delta_{AB}\delta(\xi-\eta)(\sigma^+)^{\alpha\beta}(\Pi_A^+(\xi))^{-1} \times \\ \times \big[\hat{T}_A(\xi) + 4i(-1)^A\theta_{A\alpha}'(\xi)\hat{\mathcal{D}}_A^\alpha(\xi)\big] ,$$

$$\{\hat{T}_A(\xi), \hat{T}_B(\eta)\}_{PB} = \text{(the same as in (28))} \qquad (35)$$

$$\{\hat{T}_A(\xi), \hat{\mathcal{D}}_B^\alpha(\eta)\}_{PB} = \text{(the same as in (29))}$$

As already explained in ref.(8), the harmonics $v_\alpha^{\pm 1/2}, u_\mu^a$, whose dynamics is described by the action $\hat{S}_{harmonic}$, are pure-gauge degrees of freedom and, therefore, their independence on the world sheet parameter ξ does not spoil the reparametrization invariance of the GS superstring. In the present hamiltonian framework reparametrization invariance is accounted for by the presence of the first-class constraints $\hat{T}_A(\xi)$ (33) in the action (34). Clearly, imposing the gauge fixing conditions $\Psi_A^a(\xi) = 0$ (cf. (24)) we get, in complete analogy with the superparticle case, that the system described by (34) reduces to the usual GS action (25) plus the decoupled $S_{harmonic}$ (6).

4. BRST CHARGE AND BFV HAMILTONIAN. The constraints $\hat{\mathcal{D}}_A^\alpha$ (32) form an open algebra, i.e. the structure "constants" in the first Poisson bracket relations (35) are actually functions of the canonical variables. Nevertheless, one can easily show (by direct inspection of the general equations in refs.[5]) that no higher order structure functions of the constraint algebra (35) do appear. Then, straightforward calculations yield the following particularly simple form of the BRST charge of the system (34) :

$$Q_{BRST} = Q_{harmonic} + Q_{string} + Q_{abelian} ;$$

$$Q_{harmonic} = i\eta_{ab}\big[\hat{D}^{ab} + \eta^{-a}\partial/\partial\eta^-{}_b - \eta^{-b}\partial/\partial\eta^-{}_a + \eta^a{}_d\partial/\partial\eta_{bd} - \eta^b{}_d\partial/\partial\eta_{ad}\big] \\ + i\eta^{+-}\big[D^{-+} - \eta^{-a}\partial/\partial\eta^{-a}\big] + i\eta^-{}_a D^{+a} , \qquad (36)$$

$$Q_{abelian} = i\partial/\partial\Lambda_{ab}\,\partial/\partial\tilde{\eta}^{ab} + i\partial/\partial\Lambda^{+-}\partial/\partial\tilde{\eta}^{-+} + i\partial/\partial\Lambda^{-a}\,\partial/\partial\tilde{\eta}^+{}_a + \sum_A\int_{-\pi}^{\pi} d\xi\Big[i\frac{\delta}{\delta\Lambda_A}\frac{\delta}{\delta\tilde{C}_A} - \frac{\delta}{\delta\Lambda_{A\alpha}}\frac{\delta}{\delta\tilde{\chi}_A^\alpha}\Big],$$

$$Q_{string} = \sum_A\int_{-\pi}^{\pi} d\xi\Big\{C_A\big[\hat{T}_A - 4i(-1)^A\big(C_A'\delta/\delta C_A + \chi_{A\alpha}'\delta/\delta\chi_{A\alpha}\big)\big] + \\ + \chi_{A\alpha}\hat{\mathcal{D}}_A^\alpha + (2\Pi_A^+)^{-1}(\chi_A\sigma^+\chi_A)\big[\delta/\delta C_A + 4i(-1)^A\theta_{A\alpha}'\delta/\delta\chi_{A\alpha}\big]\Big\} . \qquad (37)$$

The variables appearing in Q_{BRST} are organized as follows :

Lagrange multiplier	ghost	antighost	constraint
$\Lambda_A(\xi)$	$c_A(\xi)$	$\tilde{c}_A(\xi)$	$\hat{\pi}_A(\xi)$
$\Lambda_{A\alpha}(\xi)$	$\chi_{A\alpha}(\xi)$	$\tilde{\chi}_A^{\alpha}(\xi)$	$\hat{\mathcal{D}}_A^{\alpha}(\xi)$
Λ_{ab}	η_{ab}	$\tilde{\eta}^{ab}$	$\hat{D}^{ab}$
Λ^{+-}	η^{+-}	$\tilde{\eta}^{-+}$	D^{-+}
Λ^{-a}	η^{-a}	$\tilde{\eta}^{+a}$	D^{+a}

In order to expose explicitly the Parisi-Sourlas OSp(1,1/2) symmetry [16 19], it is useful to perform the following unitary transformation (cf. [17]) :

$$Q'_{BRST} = U\, Q_{BRST}\, U^{-1} \,,$$
$$\ln U = -(\ln\alpha)\left[c_0^{(+)}\partial/\partial c_0^{(+)} + \tilde{c}_0^{(+)}\partial/\partial\tilde{c}_0^{(+)} - 1\right] , \qquad (38)$$

$$\alpha \equiv 2\pi\left(2\lambda_0^{(+)}\right)^{1/2} , \quad \overset{(\sim)}{c}{}_0^{(\pm)} = \tfrac{1}{2}\left(\overset{(\sim)}{c}_{01} \pm \overset{(\sim)}{c}_{02}\right) , \quad \lambda_0^{(\pm)} = \tfrac{1}{2}\left(\lambda_{01} \pm \lambda_{02}\right) , \qquad (39)$$

where $\overset{(\sim)}{c}_{0A}$, λ_{0A} (A=1,2) are the zero-modes of $\overset{(\sim)}{c}_A(\xi)$, $\Lambda_A(\xi)$, respectively. With the transformation (38) Q_{string} (37) takes the form ($Q_{harmonic}$ (36) remaining unchanged) :

$$Q'_{string} = \alpha^{-1}\left[c_0^{(+)}\left(\sum_A \mathcal{L}_{0A}\right) + i\left(c_0^{(+)}\partial/\partial c_0^{(+)} - 1/2\right)\partial/\partial\tilde{c}_0^{(+)}\right] + \overline{Q} \qquad (40)$$
$$-\, 1/\pi\, c_0^{(-)}\left(\sum_A (-1)^A \mathcal{L}_{0A}\right) + 1/2\left(\sum_A \mathcal{N}_A\right)\alpha\,\partial/\partial c_0^{(+)} - 1/2\left(\sum_A (-1)^A \mathcal{N}_A\right)\partial/\partial c_0^{(-)} ,$$

where all contributions containing the zero-modes of $c_{0A}(\xi)$ are explicitly separated. In (40) the following notations were used:

$$\mathcal{L}_{0A} = \pi\int_{-\pi}^{\pi} d\xi\left[\hat{\pi}_A - 4i(-1)^A\left(c'_A\,\delta/\delta c_A + \chi'_{A\alpha}\,\delta/\delta\chi_{A\alpha}\right)\right] , \qquad (41)$$

$$\mathcal{L}_{01} = 4\pi\left(L_0 + L_0^{(ghost)}\right) , \quad \mathcal{L}_{02} = 4\pi\left(\tilde{L}_0 + \tilde{L}_0^{(ghost)}\right)$$

(zeroth Virasoro generators [1]),

$$\mathcal{N}_A = (4\pi)^{-1}\int_{-\pi}^{\pi} d\xi\,\left(\Pi_A^{+}\right)^{-1}\left(\chi_A\sigma + \chi_A\right) ,$$

and $\overline{Q}$ collects all terms of Q_{string} (37) which do not contain $c_0^{(\pm)}$, $\partial/\partial c_0^{(\pm)}$.

Then, for the BFV hamiltonian [5] :

$$H_{BFV} = \{Q_{BRST}, Y\}$$

with the specific choice $Y = \partial/\partial c_0^{(+)}$ for the BFV gauge-fixing function, we get :

$$H_{BFV} = 1/\alpha\left[\sum_A \mathcal{L}_{0A} + i\,\partial/\partial c_0^{(+)}\,\partial/\partial\tilde{c}_0^{(+)}\right] . \qquad (42)$$

In order to expose the OSp(1,1/2) symmetry of H_{BFV} let us introduce into (42) the following mode expansions :

$$X^{\mu}(\xi) = x^{\mu} + (\pi^{-1/2})\sum_{n=1}^{\infty} n^{-1/2}\left[y_n^{\mu}\cos(n\xi) + i\,\partial/\partial\tilde{y}_{n\mu}\sin(n\xi)\right] ,$$

$$\delta/\delta X_{\mu}(\xi) = \frac{1}{2\pi}\partial/\partial x_{\mu} + (\pi^{-1/2})\sum_{n=1}^{\infty} n^{-1/2}\left[\partial/\partial y_{n\mu}\cos(n\xi) + i\,\tilde{y}_n^{\mu}\sin(n\xi)\right] ;$$

$$c_A(\xi) = c_{0A} + \sqrt{2}\sum_{n=1}^{\infty}\left[b_{An}\cos(n\xi) - \partial/\partial\tilde{b}_{An}\sin(n\xi)\right] ,$$

$$\delta/\delta c_A(\xi) = \frac{1}{2\pi}\partial/\partial c_{0A} + (\pi\sqrt{2})^{-1}\sum_{n=1}^{\infty}\left[\partial/\partial b_{An}\cos(n\xi) - \tilde{b}_{An}\sin(n\xi)\right] ;$$

$$\Theta_{A\alpha}(\xi) = \Theta_{A0\alpha} + \sqrt{2}\sum_{n=1}^{\infty}\left[\varphi_{An\alpha}\cos(n\xi) - \partial/\partial\tilde{\varphi}_{An}^{\alpha}\sin(n\xi)\right] ,$$

$$\delta/\delta\Theta_{A\alpha}(\xi) = \frac{1}{2\pi}\partial/\partial\Theta_{A0\alpha} + (\pi\sqrt{2})^{-1}\sum_{n=1}^{\infty}\left[\partial/\partial\varphi_{An\alpha}\cos(n\xi) - \tilde{\varphi}_{An}^{\alpha}\sin(n\xi)\right] ;$$

$$\chi_{A\alpha}(\xi) = \chi_{A0\alpha} + \sqrt{2}\sum_{n=1}^{\infty}\left[\omega_{An\alpha}\cos(n\xi) + i\,\partial/\partial\tilde{\omega}_{An}^{\alpha}\sin(n\xi)\right] , \qquad (43)$$

$$\delta/\delta\chi_{A\alpha}(\xi) = \frac{1}{2\pi}\partial/\partial\chi_{A0\alpha} + (\pi\sqrt{2})^{-1}\sum_{n=1}^{\infty}\left[\partial/\partial\omega_{An\alpha}\cos(n\xi) + i\,\tilde{\omega}_{An}^{\alpha}\sin(n\xi)\right] .$$

The result for H_{BFV} (42) is (recall $p^2 = p^{\mu}p_{\mu}$) :

$$\alpha H_{BFV} = p^2 + i\,\partial/\partial c_0^{(+)}\,\partial/\partial\tilde{c}_0^{(+)} + 2\pi\int_{-\pi}^{\pi} d\xi\left[\left(\mathcal{P}^a\mathcal{P}_a - (4\pi^2)^{-1}p^a p_a\right) + \right.$$
$$\left. + X'^{a}X'_a + i\sum_A (-1)^A\,\Psi_A^{a}\,\Psi'_{Aa}\right] - 4\pi\sum_{n=1}^{\infty} n\left\{\left[y_n^{+}y_n^{-} + \tilde{y}_n^{+}\tilde{y}_n^{-} + i\sum_A (-1)^A b_{An}\tilde{b}_{An}\right]\right.$$
$$-\left[\partial/\partial y_n^{+}\,\partial/\partial y_n^{-} + \partial/\partial\tilde{y}_n^{+}\,\partial/\partial\tilde{y}_n^{-} + i\sum_A (-1)^A\,\partial/\partial\tilde{b}_{An}\,\partial/\partial b_{An}\right] + \qquad (44)$$
$$+ \sum_A (-1)^A\left[\left(\omega_{An\alpha}\tilde{\omega}_{An}^{\alpha} + i\,\varphi_{An\alpha}\tilde{\varphi}_{An}^{\alpha}\right) - \right.$$
$$\left.\left. - \left(\partial/\partial\omega_{An\alpha}\,\partial/\partial\tilde{\omega}_{An}^{\alpha} + i\,\partial/\partial\tilde{\varphi}_{An}^{\alpha}\,\partial/\partial\varphi_{An\alpha}\right)\right]\right\} .$$

In the mode sum on the r.h.s. of Eq.(44) one immediately observes the manifest OSp(1,1/2) symmetry in the subspaces of the mode coordinates spanned by :

$$(y^+_n, y^-_n; \tilde{b}_{1n}, b_{1n}) \;,\; (\tilde{y}^+_n, \tilde{y}^-_n; b_{2n}, \tilde{b}_{2n}) \;,$$
$$(\omega_{An\alpha}, \tilde{\omega}^\alpha_{An}; \varphi_{An\alpha}, \tilde{\varphi}^\alpha_{An}) \;. \qquad (45)$$

This OSp(1,1/2) symmetry will play an important role in the super-Poincare covariant second quantization of the GS superstring. Let us stress the crucial role of the harmonic variables (3) in isolating in a Lorentz covariant way the OSp(1,1/2)-covariant modes of $X^\mu(\xi)$ in (45) : $\overset{(\sim)}{y}{}^{\pm}_n = (v^{\mp 1/2} \sigma^\mu v^{\pm 1/2}) \overset{(\sim)}{y}_{n\mu}$.

In conclusion, let us point out that, starting with H_{BFV} (44) and following ref.[18], one can straightforwardly write down manifestly space-time supersymmetric free BRST field action for the GS superstring in terms of unconstrained "ghost-haunted" string superfield :

$$\Phi = \Phi\left[X^\mu(\cdot), \theta_{A\alpha}(\cdot), \Psi^K_A(\cdot); v^{\pm 1/2}_\alpha, u^a_\mu; \tau, \mathcal{Z}\right] .$$

Here τ denotes the evolution parameter (proper time), $\Psi^K_A(\xi)$ was defined in (17) and $\mathcal{Z}$ collectively denotes all variables (Lagrange multipliers, ghosts and antighosts) from the Table above. This topic will be discussed in more detail in a forthcomming note.

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